

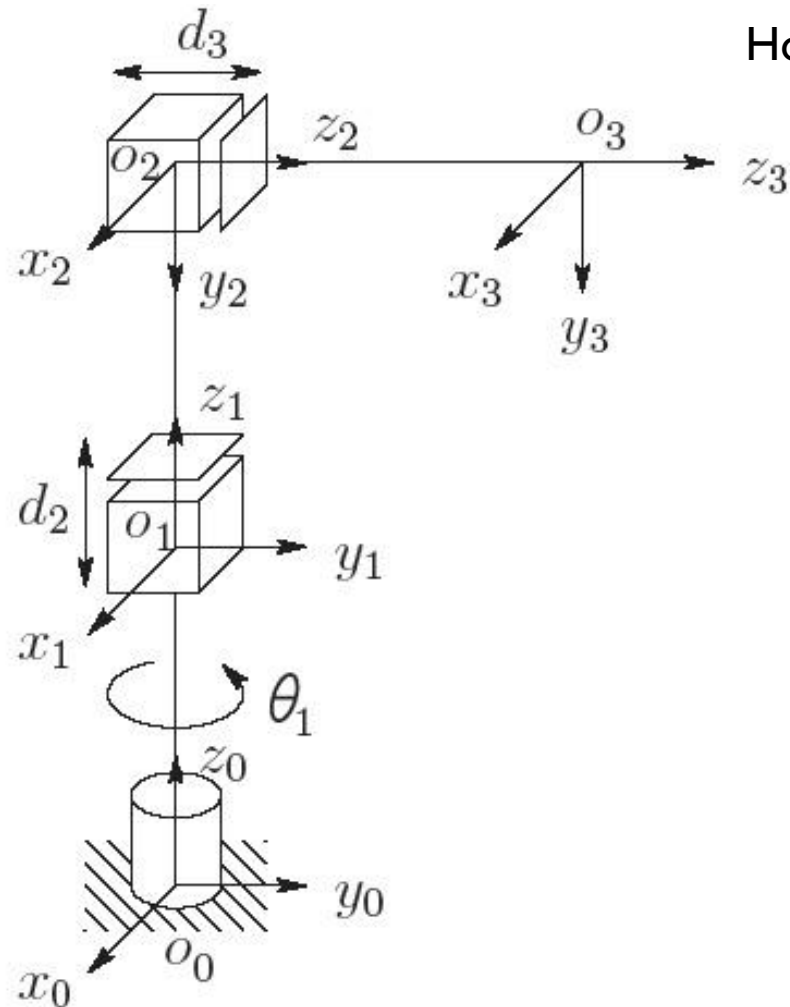
Day 08

Denavit-Hartenberg

Denavit-Hartenberg Forward Kinematics

- ▶ **RPP cylindrical manipulator**
 - ▶ <http://strobotics.com/cylindrical-format-robot.htm>

Denavit-Hartenberg Forward Kinematics



How do we place the frames?

Figure 3.7: Three-link cylindrical manipulator.

Step 1: Choose the z-axis for each frame

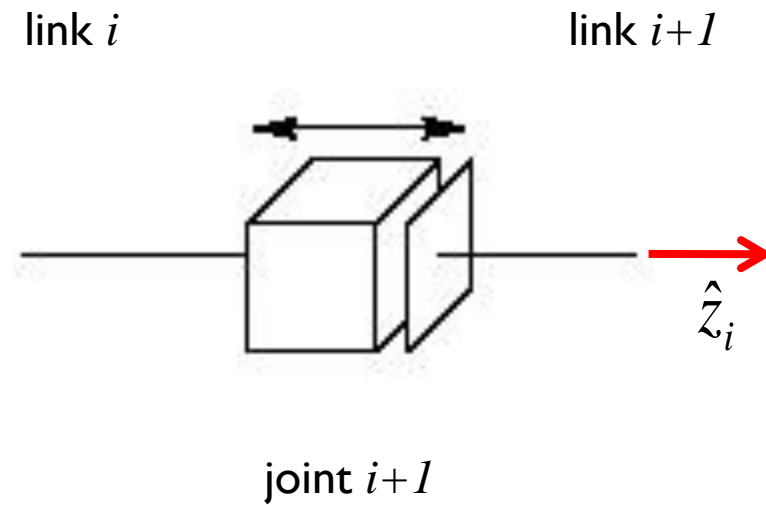
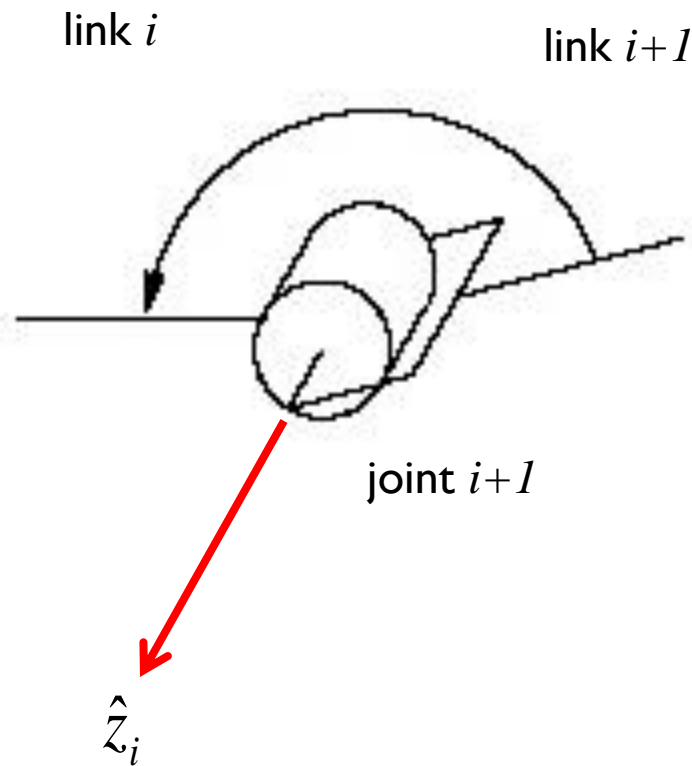
- ▶ recall the DH transformation matrix

$$T_i^{i-1} = R_{z,\theta_i} T_{z,d_i} T_{x,a_i} R_{x,\alpha_i}$$
$$= \begin{bmatrix} \boxed{\begin{matrix} c_{\theta_i} & -s_{\theta_i} c_{\alpha_i} & s_{\theta_i} s_{\alpha_i} \\ s_{\theta_i} & c_{\theta_i} c_{\alpha_i} & -c_{\theta_i} s_{\alpha_i} \\ 0 & s_{\alpha_i} & c_{\alpha_i} \end{matrix}} & \begin{matrix} a_i c_{\theta_i} \\ a_i s_{\theta_i} \\ d_i \end{matrix} \\ \begin{matrix} 0 \\ 0 \\ 0 \end{matrix} & 1 \end{bmatrix}$$

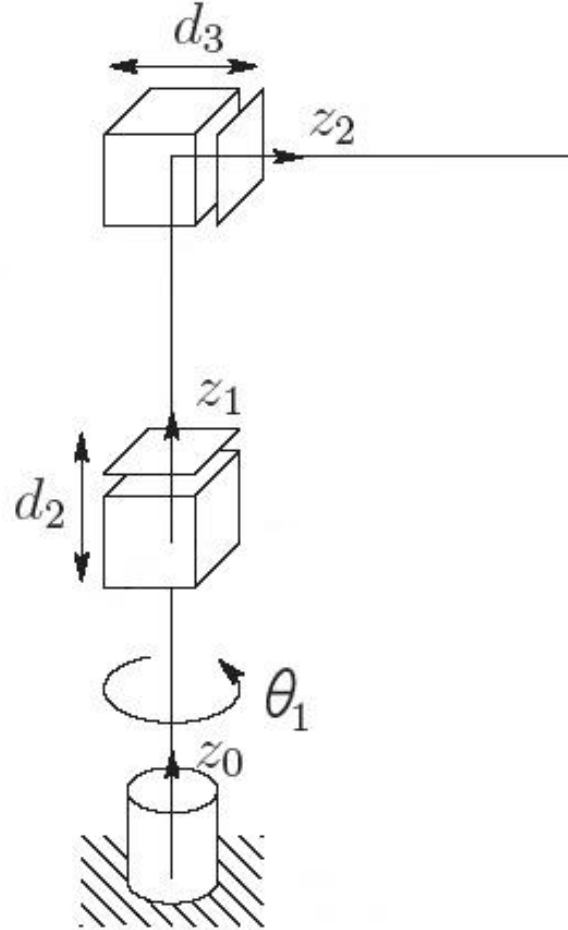
\hat{x}_i^{i-1} \hat{y}_i^{i-1} \hat{z}_i^{i-1}

Step 1: Choose the z-axis for each frame

- ▶ $\hat{z}_i \equiv$ axis of actuation for joint $i+1$



Step 1: Choose the z-axis for each frame

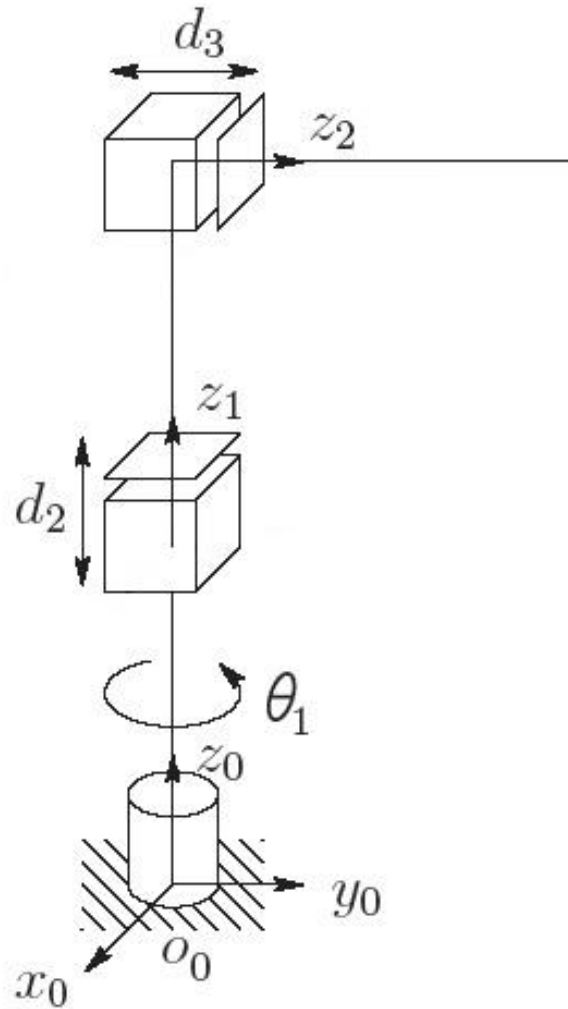


- ▶ Warning: the picture is deceiving. We do not yet know the origin of the frames; all we know at this point is that each z_i points along a joint axis

Step 2: Establish frame $\{0\}$

- ▶ place the origin o_0 anywhere on z_0
 - ▶ often the choice of location is obvious
- ▶ choose x_0 and y_0 so that $\{0\}$ is right-handed
 - ▶ often the choice of directions is obvious

Step 2: Establish frame {0}



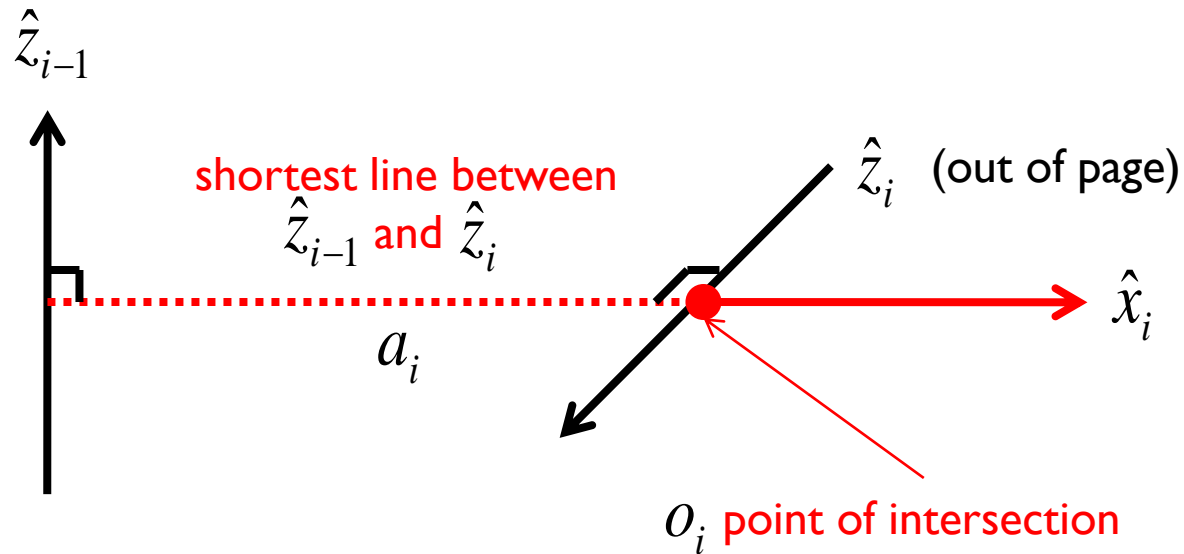
Step 3: Iteratively construct $\{1\}, \{2\}, \dots, \{n-1\}$

- ▶ using frame $\{i-1\}$ construct frame $\{i\}$
 - ▶ DH1: x_i is perpendicular to z_{i-1}
 - ▶ DH2: x_i intersects z_{i-1}
- ▶ 3 cases to consider depending on the relationship between z_{i-1} and z_i

Step 3: Iteratively construct $\{1\}, \{2\}, \dots, \{n-1\}$

▶ Case I

- ▶ z_{i-1} and z_i are not coplanar (skew)

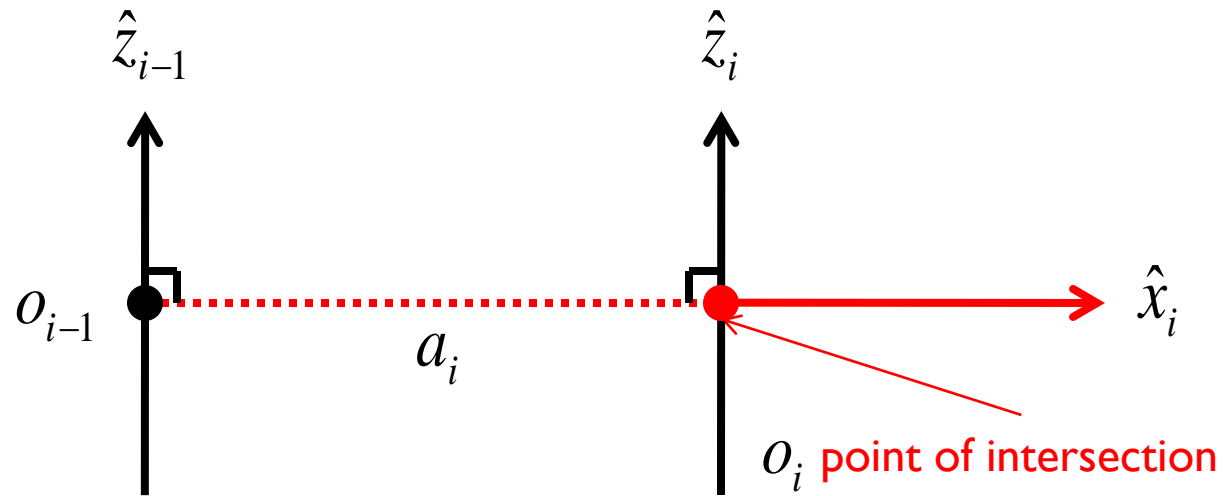


- ▶ α_i angle from z_{i-1} to z_i measured about x_i

Step 3: Iteratively construct $\{1\}, \{2\}, \dots, \{n-1\}$

▶ Case 2

- ▶ z_{i-1} and z_i are parallel ($\alpha_i = 0$)

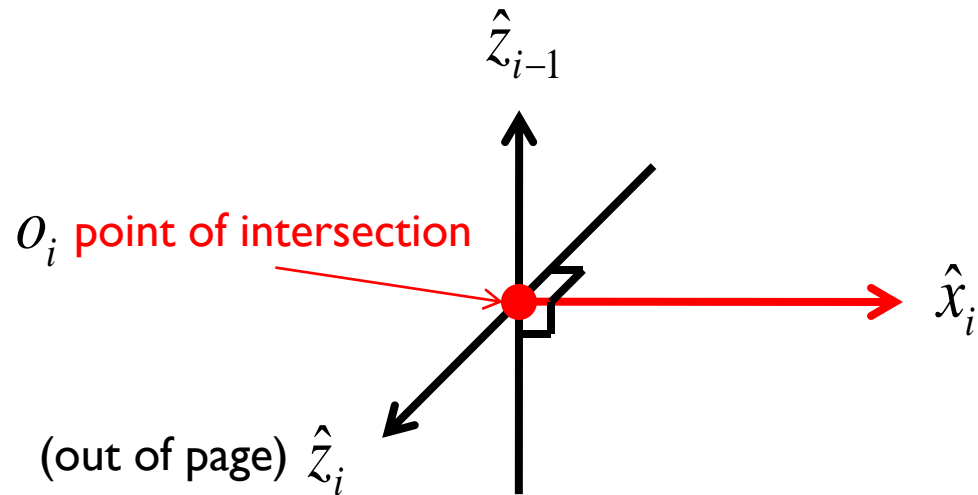


- ▶ notice that this choice results in $d_i = 0$

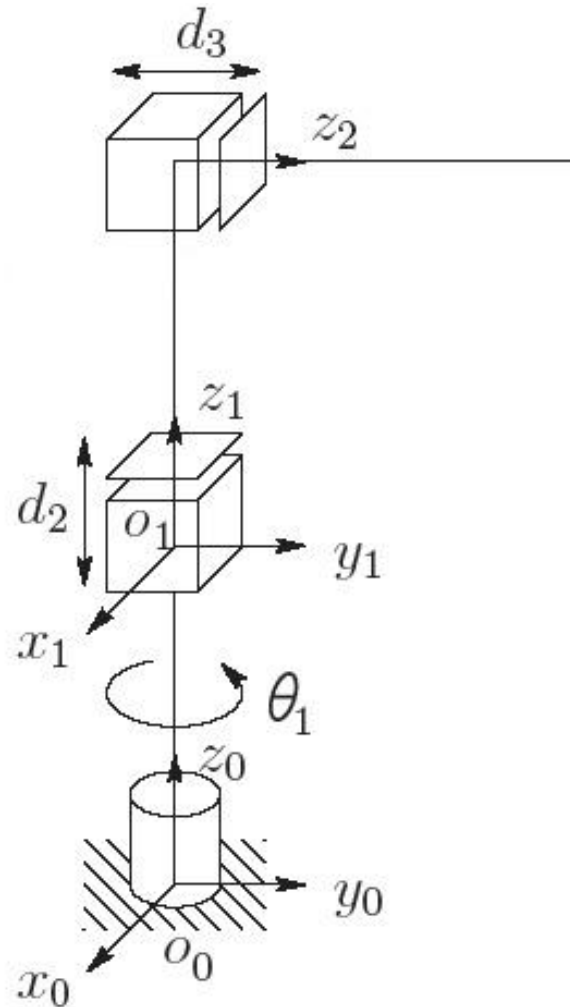
Step 3: Iteratively construct $\{1\}, \{2\}, \dots, \{n-1\}$

▶ Case 3

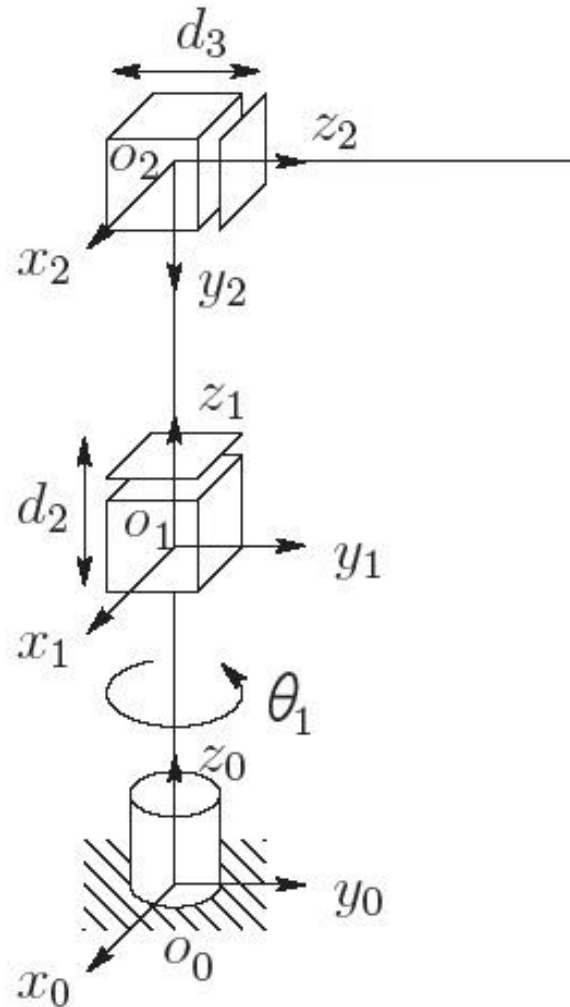
- ▶ z_{i-1} and z_i intersect ($a_i = 0$)



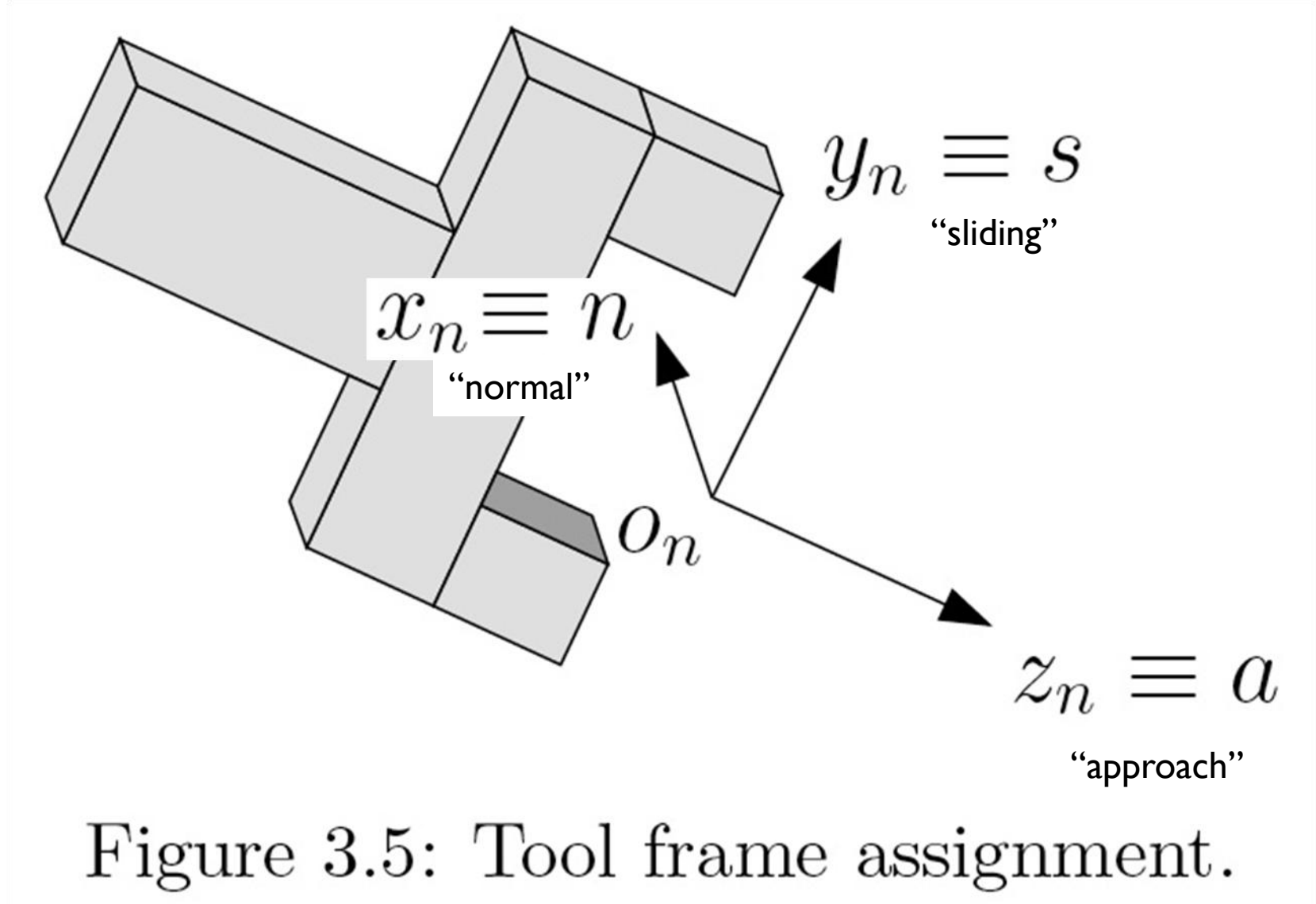
Step 3: Iteratively construct $\{1\}$, $\{2\}$, ... $\{n-1\}$



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Step 4: Place the end effector frame



Step 4: Place the end effector frame

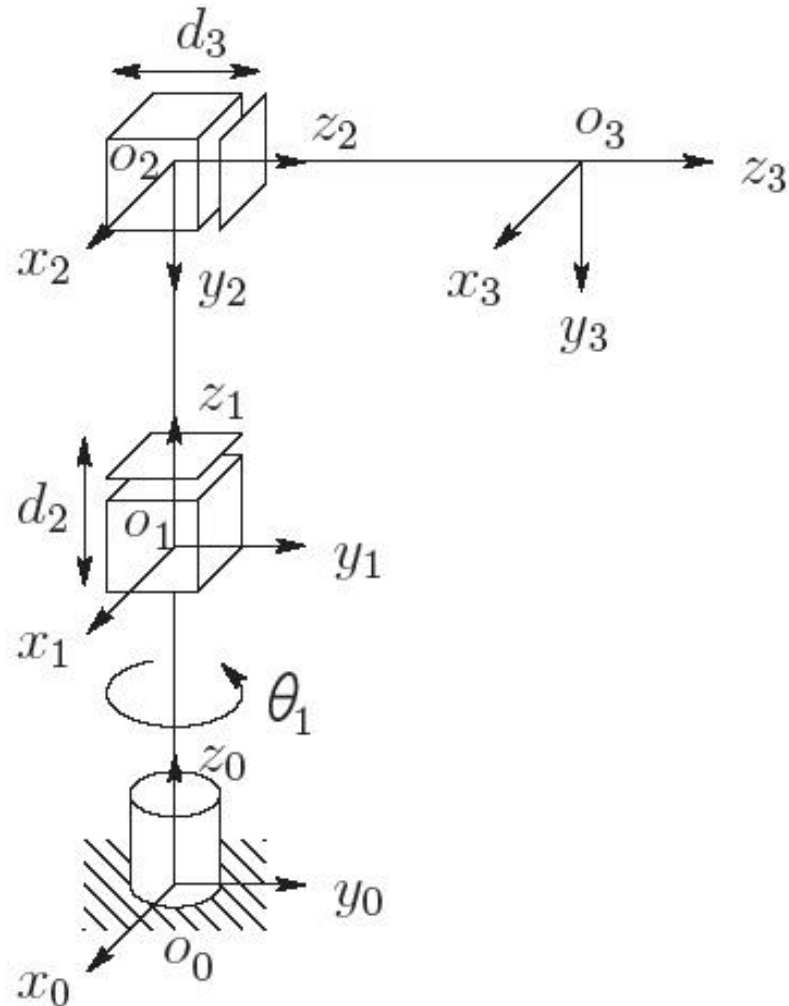
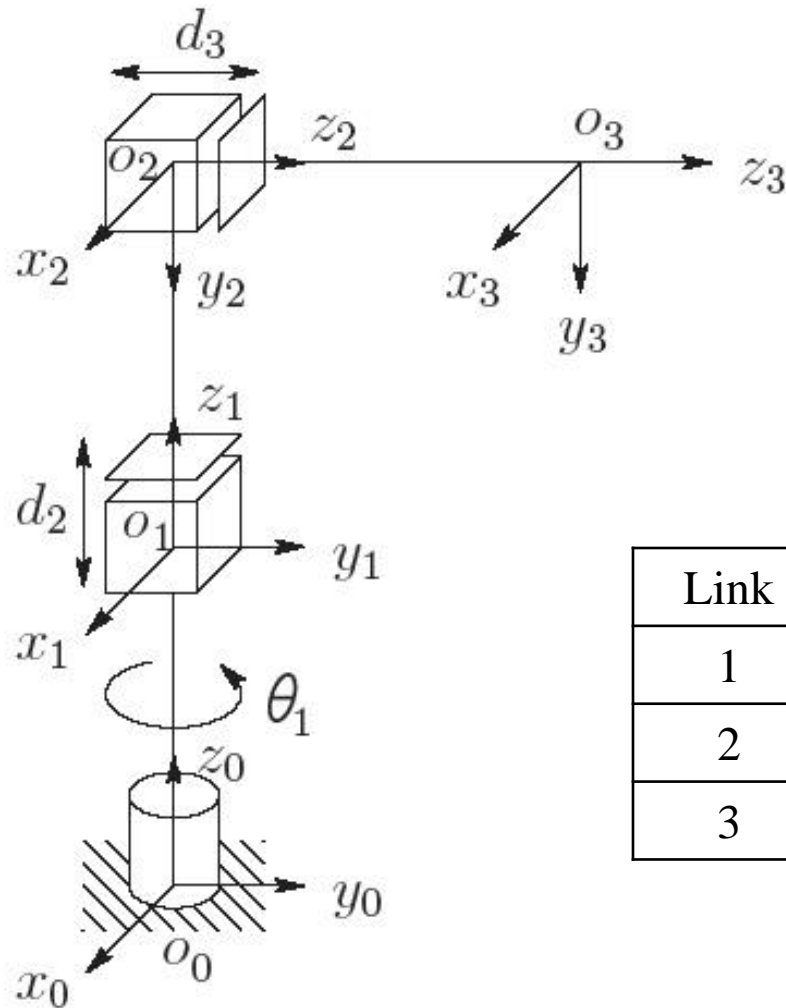


Figure 3.7: Three-link cylindrical manipulator.

Step 5: Find the DH parameters

- ▶ a_i : distance between z_{i-1} and z_i measured along x_i
- ▶ α_i : angle from z_{i-1} and z_i measured about x_i
- ▶ d_i : distance between o_{i-1} to the intersection of x_i and z_{i-1} measured along z_{i-1}
- ▶ θ_i : angle from x_{i-1} and x_i measured about z_{i-1}

Step 5: Find the DH parameters



Link	a_i	α_i	d_i	θ_i
1	0	0	d_1	θ_1^*
2	0	-90	d_2^*	0
3	0	0	d_3^*	0

* joint variable

Figure 3.7: Three-link cylindrical manipulator.